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TEACHER TIMETABLING AND WORKLOAD ALLOCATION IN NEW SECONDARY EDUCATION SYSTEM

ALOCAÇÃO DE HORÁRIOS E CARGA HORÁRIA DE PROFESSORES NO NOVO ENSINO MÉDIO

ASIGNACIÓN DE HORARIOS Y CARGA HORARIA DOCENTE EN EL NUEVO SISTEMA DE BACHILLERATO

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ABSTRACT

The Brazilian new secundary education reform (Law No. 13,415/2017) introduced curricular and structural transformations that significantly increased the complexity of teacher workload allocation. School administrators now confront interdependent challenges, including teacher availability, balanced distribution of operational workloads, commuting constraints, and limited physical infrastructure (e.g., "classrooms," "laboratories," "IT infrastructure") within a more flexible curriculum. Teacher timetabling is a classical NP-hard problem in Operations Research, and Mixed-Integer Linear Programming (MILP) provides a rigorous framework to model such interdependent constraints. This study proposes and implements an MILP optimization model for teacher workload allocation in a large public secundary educationin São José dos Pinhais, Paraná, Brazil. The real-world instance included 49 teachers and 19 classes, totaling 745 weekly hours. The formulation explicitly incorporates institutional constraints—such as daily workload limits, asynchronous class integration, and the shared-resource restriction for Physical Education—as well as teacher preferences regarding paired lessons and activity hours (HA) grouping. The model, implemented in Julia (JuMP) and solved with the Gurobi Optimizer, achieved an exact optimal solution in approximately 35 seconds. The optimization resulted in reducing one teacher's weekly presence by a full day while maintaining the same number of working days for others. Furthermore, it eliminated the need for manual scheduling adjustments, automatically ensuring consistent allocation of asynchronous lessons and the non-overlapping use of the school's sports court. By combining operational efficiency with adherence to teacher preferences, the proposed MILP formulation proved to be computationally tractable and applicable to real educational contexts. The model serves as a decision-support tool that enhances school organization and improves the overall management of teaching schedules.

Keywords: Timetabling. Mixed-Integer Linear Programming. Optimization. Brazilian New Secundary Education.

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RESUMO
A reforma do Novo Ensino Médio no Brasil (Lei n.º 13.415/2017) introduziu mudanças curriculares e estruturais que aumentaram significativamente a complexidade da alocação da carga horária docente. Os gestores escolares passaram a enfrentar desafios como disponibilidade de professores, equilíbrio da carga de trabalho, restrições de deslocamento e limitações de infraestrutura em um currículo mais flexível e diversificado. A alocação de horários constitui um problema clássico e NP-difícil em Pesquisa Operacional, sendo a Programação Linear Inteira Mista (MILP) uma formulação matemática adequada para representar restrições interdependentes. Este estudo propõe e implementa um modelo MILP para otimizar a alocação da carga horária em uma escola pública de Ensino Médio em São José dos Pinhais, cidade do estado do Paraná, Brasil. O problema real envolveu 49 professores e 19 turmas, totalizando 745 horas semanais. A formulação incorpora restrições institucionais como limites diários de jornada, integração de aulas assíncronas e uso compartilhado de recursos para Educação Física, além de preferências docentes quanto à geminação de aulas e agrupamento das Horas de atividades (HA). O modelo, desenvolvido em Julia (JuMP) e resolvido com o Gurobi Optimizer, alcançou solução ótima exata em cerca de 35 segundos. A otimização reduziu um dia presencial na instituição para um docente e suprimiu a necessidade ajustes manuais de horário, uma vez que o cronograma gerado passou a satisfazer automaticamente a todas as restrições institucionais e operacionais. A formulação proposta mostrou-se computacionalmente eficiente e aplicável em contextos educacionais reais, funcionando como ferramenta de apoio à decisão para aprimorar a gestão da jornada docente e a organização escolar.

Palavras-chave: Dimensionamento de Horários. Programação Linear Inteira Mista. Otimização. Novo Ensino Médio.

RESUMEN
La reforma del Nuevo Bachillerato em Brasil (Ley N.º 13.415/2017) introdujo cambios curriculares y estructurales que aumentaron significativamente la complejidad en la asignación de la carga horaria docente. Los gestores escolares enfrentan desafíos como la disponibilidad de profesores, el equilibrio de la carga de trabajo, las restricciones de desplazamiento y las limitaciones de infraestructura dentro de un currículo más flexible y diversificado. La asignación de horarios constituye un problema clásico y NP-difícil en la Investigación Operativa, siendo la Programación Lineal Entera Mixta (MILP) una formulación matemática adecuada para representar restricciones interdependientes. Este estudio propone e implementa un modelo MILP para optimizar la asignación de la carga horaria en una escuela pública de educación media en São José dos Pinhais (PR), Brasil. La instancia real involucró 49 docentes y 19 clases, totalizando 745 horas semanales. La formulación incorpora restricciones institucionales como los límites diarios de jornada, la integración de clases asincrónicas y el uso compartido de recursos para Educación Física, además de las preferencias docentes relacionadas con la geminación de clases y el agrupamiento de Horas de Actividad. El modelo, desarrollado en Julia (JuMP) y resuelto con el Gurobi Optimizer, alcanzó una solución óptima exacta en aproximadamente 35 segundos. La optimización redujo un día completo de presencia semanal para un docente y eliminó la necesidad de ajustes manuales en la programación. Así, la formulación propuesta demostró ser computacionalmente eficiente y aplicable en contextos educativos reales, funcionando como una herramienta de apoyo a la decisión para mejorar la gestión del trabajo docente y la organización escolar.

Palabras clave: Asignación de Horários. Programación Lineal Entera Mixta. Optimización. Nuevo Bachillerato em Brasil.

1 INTRODUCTION

The Brazilian educational system has undergone multiple transformations throughout the 20th century, induced by political, social, and economic vectors (Codes, Fonseca and Araújo, 2021). For an extensive period, secondary education maintained a rigid structure, characterized by standardized curricula and limited flexibility to accommodate the diversity of student interests and abilities. This model proved insufficient in the face of contemporary demands, notably technological advancement, globalization, and the new requirements of the labor market.

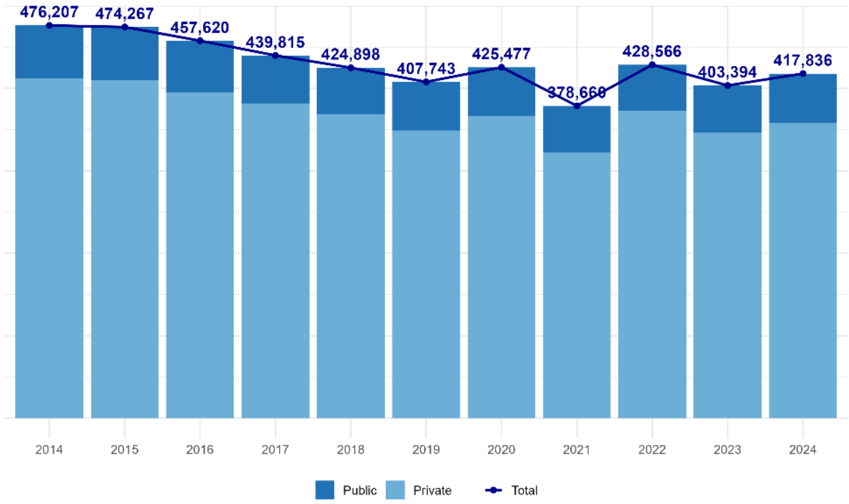
The Federal Constitution of 1988 established education as a fundamental right and a State duty, constituting a milestone for the democratization of access. Nonetheless, the persistence of structural challenges—such as educational inequality, the unsatisfactory quality of learning, and high dropout rates—made restructuring imperative. The National Education Plan (PNE), instituted in 2001 and revised in 2014, outlined goals for the expansion and improvement of basic education. However, the secundary education reform only gained centrality in 2017 with the enactment of Law No. 13,415 (Brasil, 2017), which established the new secundary education.

This reform introduced substantial modifications, increasing curricular flexibility and organizing secondary education into two complementary axes: the General Basic Education (FGB), comprising 1,800 hours, and the Formative Itineraries, totaling 1,200 hours. The implementation of this new architecture in the state of Paraná, Brazil was initiated in 2019, culminating in the development of the curricular reference for secundary education, with support from the Program to Support the Implementation of the National Common Curricular Base (ProBNCC). The ProBNCC, a collaborative initiative led by the Department of Education of Brazil, known nationally as the Ministry of Education (MEC), the National Council of Education Secretaries (CONSED), and the National Union of Municipal Education Directors (UNDIME), aims to provide technical, pedagogical, and financial support to federative entities in the process of curricular alignment with the BNCC.

According to the 2024 School Census (INEP, 2024), the state of Paraná, Brazil recorded 417,836 secundary education enrollments, of which 85.7% corresponded to the public network (358,012 students) and 14.3% to the private network (59,824 students). The teaching staff remained predominantly concentrated in the state system, and night-shift education continued to play a significant role, especially in serving students who balance study and work. This scenario intensifies the challenges of educational management, particularly in large public schools, where the organization of teacher workload becomes a critical factor for ensuring both operational efficiency and the quality of the teaching–learning process.

Figure 1

Evolution of secundary education enrollments in Paraná (2014–2024)



Source: Prepared by the authors themselves.

The efficient allocation of teachers—conceptualized in the literature as teacher workload allocation or teacher timetabling—is recognized as a classic problem in Operations Research, frequently modeled as an NP-hard combinatorial optimization task due to its multi-constraint nature (Schaerf, 1999). This task involves addressing multiple constraints, including teacher availability, maximum daily class limits, individual preferences, curricular requirements, and, in certain instances, commute costs. Numerous studies demonstrate the effectiveness of Mixed-Integer Linear Programming (MILP) models. Gu et al. (2025) conducted a systematic review of optimization approaches for academic timetabling, highlighting the predominance of MILP techniques and discussing their practical implementation challenges in real-world educational settings.

However, a gap persists in applications focused specifically on the context of the Brazilian new secundary education, considering it curricular and organizational specificities.

The analysis of the evolution of the public secundary education teaching staff in Paraná, presented in Figure 2, reveals significant fluctuations in the number of teachers over the last decade. In 2014, the state had 32,849 teachers, a figure that gradually declined until reaching 27,936 in 2021. From 2022 onwards, a recovery is observed, with the number increasing to 30,682 in 2023 and stabilizing at 29,949 in 2024. This dynamic reflects both the impact of management and hiring policies and the need for constant adjustments in teacher allocation to keep pace with changes in student demand.

Figure 2

Total teaching staff in Paraná's public schools (2014–2024)

Source: Prepared by the authors themselves.

In addition to the pre-existing structural challenges, the geographic characteristics of São José dos Pinhais, a city located in the state of Paraná, southern Brazil, impose further complexities upon teacher allocation. The municipality exhibits a considerable territorial extension, with peripheral areas situated at significant distances from the urban center and from one another. Figure 3 illustrates this spatial dispersion via a geospatial map. The map's central blue marker denotes the location of the school unit, clearly evidencing the substantial distances from various other neighborhoods given the city's overall territorial spread. A substantial fraction of the teaching staff does not reside within the same administrative region as their assigned educational unit, which consequently elevates both commuting time and associated costs. Within this framework, the implementation of an optimized allocation strategy aimed at minimizing the number of physical presence days at the school unit acquires elevated relevance, as it promotes a direct reduction in the workload burden on educators while ensuring strict compliance with institutional requirements for minimum mandatory hours.

Figure 3

Geospatial map of São José dos Pinhais

Source: São José Dos Pinhais, 2025

Addressing these challenges, the present study proposes and implements a Mixed-Integer Linear Programming mathematical model for the optimization of teacher workload allocation within a large-scale public school system unit located in São José dos Pinhais, a city located in the state of Paraná, Brazil. The model is designed to maximize operational efficiency in class distribution by strictly enforcing the predefined workloads of each faculty member. Concurrently, the formulation seeks the minimization of the number of teacher physical presence days and the accommodation of the specific operational demands of the evening shift.

2 MATERIAL AND METHODS

This study adopts an applied and descriptive design grounded in mathematical modeling via Mixed-Integer Linear Programming. The primary objective was to propose and implement an optimization model for teacher workload allocation in a large public state educational unit located in the municipality of São José dos Pinhais, Paraná. The model was conceptualized to incorporate the institutional, pedagogical, and logistical constraints inherent to the context of implementing the New Brazilian Secondary Education curriculum. In the current operational scenario, the administrative staff utilizes dedicated software to allocate synchronous lessons and activity hours (HA)—the legally mandated portion of a teacher's workload dedicated to extraclass pedagogical activities—while the distribution of asynchronous lessons is performed manually. This hybrid process underscores the practical necessity for a fully integrated optimization model.

The research was conducted at School A, the municipality has a population of 329,628 inhabitants, according to the 2022 population census conducted by the Brazilian Institute of Geography and Statistics (IBGE, 2024). School A primarily serves a student population residing in peripheral neighborhoods, characterized by a reliance on public transportation. This logistical factor reinforces the relevance of optimizing class allocation, particularly for the evening shift.

Furthermore, School A possesses only one sports court, a resource shared among all Physical Education classes. Under the current allocation process, this constraint is not automatically managed by the scheduling software. Instead, adjustments are performed manually by the administrative staff to prevent the simultaneous assignment of more than two Physical Education teachers. While the simultaneous allocation of up to two teachers is considered tolerable, the presence of three or more classes at the same time generates logistical difficulties. This challenge is particularly relevant as the school employs four Physical Education teachers. Although manual corrections are intended to respect the resource limitation, they require considerable effort and increase the likelihood of inconsistencies. Embedding this constraint directly into the MILP model eliminates the need for repetitive manual interventions during the optimization process.

The study encompassed 49 teachers and 19 classes, totaling 69 curricular components distributed throughout the school week. Each class maintained a fixed workload of 30 weekly lessons, corresponding to six lessons per day. In accordance with School A’s operational guidelines, five daily lessons were conducted synchronously, while the sixth lesson of the evening shift was designated to occur asynchronously.

The study population consists of all 49 faculty members at School A who taught the evening shift during 2024, and a census method was used, meaning the entire population was included in the study, not just a sample. This approach is also referred to as a census because it surveys every member of the group under study.

It is pertinent to note that each teacher enters the allocation process with a predefined workload, specifying the number of lessons and the classes they are assigned to teach. This initial designation establishes a fundamental pedagogical condition: the same teacher must not instruct more than one subject within a single class.

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For each teacher, detailed availability information was collected, specified both by days of the week and by individual time slots (lesson level), based on two criteria: logistical constraints and personal preferences. The teacher was allowed to designate time slots as unavailable due to logistical reasons (e.g., external commitments, multiple school assignments) or personal preference, reflecting days when they would prefer not to teach.

This level of detail is essential, as it ensures that critical constraints, such as the inability to be assigned to the first lesson of the day due to commuting distance and traffic conditions, can be strictly adhered to. Furthermore, the inclusion of individual preferences in the modeling process grants greater professional value to the educators, aligning institutional planning with the teachers' needs and well-being.

The MILP model is designed to accommodate the practice of a single teacher instructing multiple distinct subjects across different classes (e.g., Mathematics, Financial Education, and Physics). However, the model incorporates the operational condition that the teacher must not be allocated to instruct different subjects within the same class. By modeling availability at the lesson level, the system thus guarantees a realistic and consistent allocation with the faculty's actual working conditions and expectations.

2.1 LINEAR PROGRAMMING (LP)

Linear Programming is a mathematical optimization technique in which both the objective function and the constraints are expressed as linear relationships. It is widely applied in various fields such as economics, engineering, logistics, and education. LP problems are characterized by the proportionality, additivity, and divisibility of resources, which allows solutions to be represented in continuous domains (Taha, 2017). A typical LP model seeks to maximize or minimize an objective function subject to a set of linear constraints, ensuring that all decision variables satisfy feasibility conditions such as non-negativity.

The classical Linear Programming (LP) problem consists of optimizing (maximizing or minimizing) a linear objective function subject to a set of linear constraints. In general, the model can be represented as follows:

$$Maximize\ (or\ Minimize)\ \ Z = \sum_{j=1}^{n} c_j x_j \qquad (1)$$

$subject\ to$:

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \ (i = 1,2, \dots, m) \quad (2)$$

$$x_j \geq 0 \ (j = 1,2, \dots, n) \quad (3)$$

- Z is the objective function, representing the value to be optimized (such as cost or profit);
- c_j are the coefficients of the objective function;
- x_j are the decision variables;
- a_{ij} are the technological coefficients of the constraints;
- b_i are the available amounts of resources;

2.2 MIXED-INTEGER LINEAR PROGRAMMING (MILP)

MILP extends LP by allowing some or all of the decision variables to take only integer values, while others may remain continuous. This formulation is particularly useful for real-world problems where decisions are discrete by nature, such as assigning teachers to classes or scheduling events in time slots. Although MILP increases the computational complexity of the problem, it offers more realistic and applicable solutions in scenarios where fractional allocations are not feasible.

It should be noted that the MILP model preserves the same structure as the classical LP model, with the addition of the integrality constraint (Equation 7).

$$Maximize\ (or\ Minimize)\ \ Z = \sum_{j=1}^{n} c_j x_j \quad (4)$$

$$subject\ to:$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq b_i \ (i = 1,2, \dots, m) \quad (5)$$

$$x_j \geq 0 \ (j = 1,2, \dots, n) \quad (6)$$

$$x_j \in Z \quad (7)$$

2.3 MATHEMATICAL MODEL FOR WORKLOAD ALLOCATION

The following mathematical model was developed to optimize teacher workload allocation, considering institutional, operational, and preference constraints. The formulation uses Mixed-Integer Linear Programming and defines the decision variables, objective function, and main constraints of the problem.

2.3.1 Parameters

P: set of Professors $(p = 1 \dots P)$, $P = 49$.

T: set of Classes $(t = 1 \dots T), T = 19$.

D: number of days in a week $(d = 1 \dots D), D = 5$.

H: number of time slots per day $(h = 1 \dots H), H = 5$.

$CH_{p,t}^{sync}$: synchronous workload of teacher p for class t.

$CH_{p,t}^{async}$: asynchronous workload of teacher p for class t.

HA_p: activity hours for teacher p.

$Disp_{p,d,h} \in \{0,1\}$: availability of teacher p on day d and time slot h. (1 available, 0 unavailable).

$PF_p^{gem} \in \{-1,0,1\}$: synchronous pairing preference of teacher p. (-1 avoid, 0 neutral, 1 prefer).

$PF_p^{HA} \in \{-1,0,1\}$: concentrated HA preference of teacher p.(-1 spread, 0 neutral, 1 group).

$PE \subseteq P$: subset of physical education professors (shared gymnasium).

2.3.2 Decision Variables

$x_{p,t,d,h} \in \{0,1\}$: 1 if teacher p teaches synchronous class t on day d and slot h, 0 otherwise.

$a_{p,t,d} \geq 0$: number of asynchronous lessons of teacher p for class t on day d.

$y_{p,d,h} \in \{0,1\}$: 1 if teacher p has HA in slot (d,h).

$v_{p,d} \in \{0,1\}$: daily assignment, 1 if teacher p works on day d.

$s_{p,t,d,h}, z_{p,t,d,h} \in \{0,1\}$: auxiliary variables for synchronous pairing (s single lesson, z block start).

$a\_slot_{p,d,h} \in \{0,1\}$: marks slots occupied by asynchronous lessons.

$r_{p,d,h} \in \{0,1\}$: marks the start of concentrated HA blocks.

2.3.3 Constraints

Teacher can only be assigned to slots where $Disp_{\{p,d,h\}} = 1$.

$$\sum_{t} x_{p,t,d,h} + y_{p,d,h} + a\_slot_{p,d,h} \leq Disp_{\{p,d,h\}}, \ \forall\, p, d, h \tag{8}$$

Synchronous workload per teacher: Each teacher must meet their total synchronous workload.

$$\sum_{d}\sum_{h} x_{p,t,d,h} = \sum_{t} CH_{p,t}^{sync}, \qquad \forall\, p, t \tag{9}$$

Asynchronous workload per teacher: Ensures all asynchronous lessons are assigned for each teacher.

$$\sum_{t} a_{p,t,d} = \ CH_{p,d}^{async}, \qquad \forall p, d \tag{10}$$

Exact HA allocation per teacher: Activity hours are assigned according to the total required.

$$\sum_{d}\sum_{h} y_{p,d,h} = \ HA_{p}, \ \ \forall p \tag{11}$$

Slot conflict per teacher: No teacher can have more than one activity in the same slot.

$$\sum_{t} x_{p,t,d,h} + y_{p,d,h} + a\_slot_{p,d,h} \leq 1\,, \forall p, d, h \tag{12}$$

Single classroom per class: Each class can have at most one teacher per time slot.

$$\sum_{p} x_{p,t,d,h} \ \leq 1, \qquad \forall t, d, h \tag{13}$$

Daily workload limit per teacher: Ensures teachers do not exceed the daily maximum of lessons and HA.

$$\sum_t \sum_h x_{p,t,d,h} + \sum_h a\_slot_{p,d,h} + \sum_h y_{p,d,h} \leq maxDaily, \qquad \forall p, d \tag{14}$$

Synchronous pairing: Models paired lesson blocks for synchronous classes.1

$$x_{p,t,d,h} = s_{p,t,d,h} + z_{p,t,d,h} + z_{p,t,d,h-1}, \qquad \forall p, t, d, 2 \leq h \leq H-1 \tag{15}$$

Concentrated HA blocks: Marks the start of concentrated HA blocks according to preferences.

$$\begin{aligned} r_{p,d,1} &= y_{p,d,1} \\ r_{p,d,h} &\geq y_{p,d,h} - y_{p,d,h-1}, \qquad h \geq 2 \end{aligned} \tag{16}$$

Physical Education (PE) Classes Constraint: This constraint ensures that no more than two PE teachers are assigned to the same time slot simultaneously, reflecting the reality that the school has only one sports court that can accommodate at most two classes at once. The restriction applies exclusively to synchronous PE classes and does not affect asynchronous classes or HA.

$$\sum_{p \in PE} x_{p,t,d,h} \leq 2, \qquad \forall d, h \tag{17}$$

Daily assignment: Defines whether a teacher is working on a given day based on assigned activities.

$$\sum_t \sum_h x_{p,t,d,h} + \sum_h a\_slot_{p,d,h} + \sum_h y_{p,d,h} \leq M.v_{p,d} \tag{18}$$

2.3.4 Objective Function

The central objective of the workload allocation model is the minimization of the total scheduling cost, which is defined by the weighted combination of operational efficiency and teacher preference satisfaction. The efficiency term aims to minimize the total number of physical presence days required from the entire staff at the educational unit, quantified by penalizing each day a teacher is scheduled, as represented by the binary variable $v_{p,d} = 1$. Additionally, the objective function integrates the preference satisfaction cost, applying penalties (λ=1.0) for any deviation from the declared scheduling preferences. In this study, all preference-related penalty weights (λ) were assigned the value of 1.0 for baseline implementation. This choice was made to maintain a neutral balance between operational efficiency and teacher preference satisfaction. However, these weights are not fixed: they can be adjusted according to institutional priorities or managerial strategies. Future research may explore the impact of varying λ values to calibrate trade-offs between reducing teacher presence days and maximizing adherence to individual scheduling preferences. These deviations include the allocation of synchronous lessons as isolated slots (single) when the teacher prefers double blocks (geminated), or vice-versa; and the allocation of HA in long blocks when the preference is for distribution throughout the day, or in short/distributed blocks when the preference is for concentration. Consequently, the model seeks the optimal trade-off that enables the reduction of overall daily presence while maximizing adherence to the personal scheduling preferences of the teaching staff.

$$min\ Z = \sum_{p}\sum_{d} v_{p,d} + \lambda_1 \sum_{p:\,PF_{p=1}^{gem}} \sum_{t}\sum_{d}\sum_{h} s_{p,t,d,h} + \lambda_2 \sum_{p:PF_{p=-1}^{gem}} \sum_{t}\sum_{d}\sum_{h} z_{p,t,d,h} + \tag{19}$$

$$+ \lambda_3 \sum_{p:PF_{p=1}^{HA}} \sum_{d}\sum_{h} r_{p,d,h} + \lambda_4 \sum_{p:PF_{p=-1}^{HA}} \sum_{d}\left(\sum_{h} y_{p,d,h} - \sum_{h} r_{p,d,h}\right)$$

The mathematical formulation is formally defined in Equation (19), subject to constraints described in Equations (8)–(18).

3 RESULT

The proposed Mixed-Integer Linear Programming (MILP) model was implemented in Julia using the JuMP package and solved with the Gurobi Optimizer (version 12.0.2). Computational experiments were executed on a machine equipped with an Intel® Core™ i5-1035G1 processor (1.00 GHz, 8 logical cores) running Windows 10. The test instance comprised 49 teachers and 19 classes, resulting in a total weekly workload of 745 hours, subdivided into 475 synchronous, 95 asynchronous, and 175 activity hours (HA). The model

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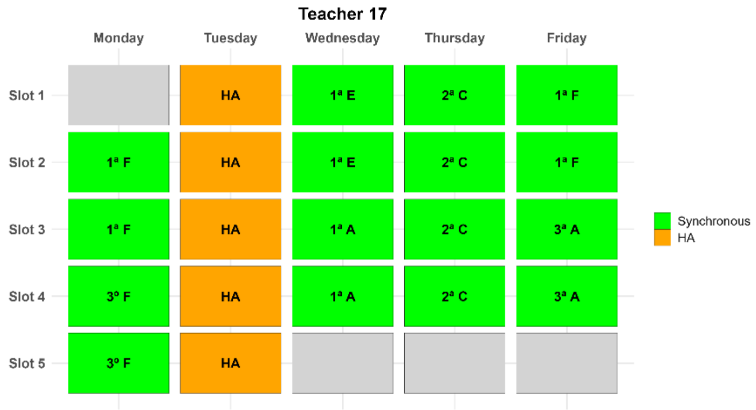
formulation included 27,447 constraints, 73,843 integer variables (69,090 of which were binary), and 277,770 non-zero coefficients. After presolve, the model was significantly reduced to 10,886 constraints and 30,806 binary variables, improving computational efficiency. Gurobi employed deterministic concurrent optimization with primal and dual simplex methods, applying several classes of cutting planes (Gomory, Cover, Clique, and Relax-and-Lift). The solver achieved the optimal solution after exploring a single branch-and-bound node and 81,824 simplex iterations, obtaining an objective value of ($Z = -21.0$) with a 0.0000 % optimality gap in approximately 35 seconds. These results confirm that the proposed formulation is computationally tractable and capable of reaching global optimality efficiently even in realistic large-scale instances of the teacher allocation problem.

Figure 4 presents the allocation for Teacher 17. This teacher expressed a preference for distributing non-HA across the week and avoiding geminated lessons. The resulting timetable clearly reflects these preferences: HA periods are spread throughout multiple days, and synchronous lessons are arranged in single, non-consecutive slots. Such an outcome highlights the model's flexibility in accommodating individual scheduling tendencies while maintaining overall workload balance and institutional feasibility.

Figure 4

Optimized timetable allocation for Teacher 17

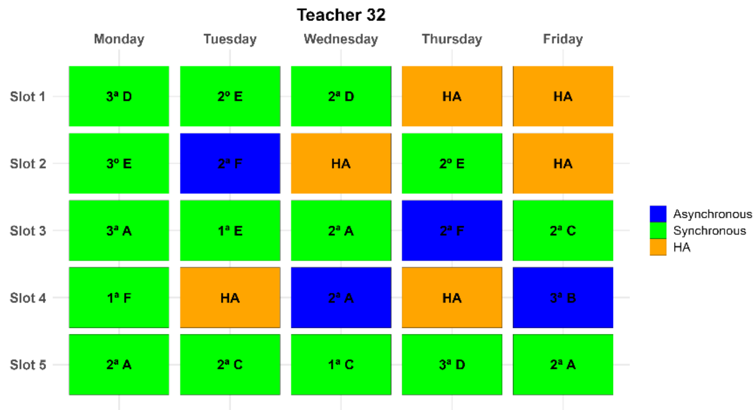


Source: Prepared by the authors themselves.

Figure 5 presents an example of a weekly schedule for a teacher 32 who prefers to distribute her non-teaching hours throughout the week rather than concentrating them in consecutive blocks. The timetable also avoids paired or geminated synchronous classes, resulting in a more balanced and diversified distribution of activities across the available time slots.

Figure 5

Optimized timetable allocation for Teacher 32



Source: Prepared by the authors themselves.

In comparation with the previus model, in which asynchronous lessons were manually distributed, the new MILP formulation fully automates the allocation process, eliminating the need for manual adjustments by the staff responsible for timetable construction. In the earlier version, asynchronous hours required post-processing corrections, often leading to discrepancies between the planned and assigned workloads. The updated model explicitly incorporates these activities, ensuring consistency between each teacher's total workload and declared availability. Furthermore, it accounts for individual teacher preferences—such as the concentration of classes on specific days and the grouping of HA—thereby improving alignment with both personal and institutional needs. As a result, one teacher's weekly workload was reduced by one day, while the others retained the same number of teaching days, reflecting a more balanced and preference-aware allocation. This new approach not only enhances operational efficiency but also fosters teacher satisfaction and well-being.

4 DISCUSSION

The results obtained in this study demonstrate that the Mixed-Integer Linear Programming model is capable of addressing the complexity of teacher workload allocation under the Brazilian new secondary education reform, while balancing operational efficiency and teacher preferences. The reduction in the number of presence days required from teachers and the automatic enforcement of the Physical Education court constraint illustrate significant managerial improvements.

Consistent with the literature (Fonseca et al., 2017; Góes, Costa, Steiner, 2010), the findings substantiate the robustness of exact approaches based on MILP, while also underscoring scalability challenges when applied to larger educational systems or multiple schools. Beyond the classical contributions, recent applications within the Brazilian New Secondary Education framework further attest to the practical relevance of exact optimization models (Vieira, Conte and Siqueira, 2024).

Despite these advances, some limitations must be acknowledged. The application was restricted to a single institution, which limits the generalization of results. Moreover, the reliance on specialized software (Julia/JuMP + Gurobi) may hinder immediate adoption by educational authorities. Another relevant point concerns solution time: while acceptable in the studied case, computational performance may become a bottleneck in larger-scale scenarios.

Future research should explore the adoption of techniques to accelerate model solving, such as:

- Problem decomposition methods (e.g., Benders, Dantzig-Wolfe), splitting the formulation into smaller subproblems;
- Hybrid algorithms combining MILP with heuristics or metaheuristics (e.g., GRASP, Genetic Algorithms, Simulated Annealing) to generate high-quality initial solutions;
- Enhanced preprocessing and cutting-plane strategies to reduce the search space;
- Use of parallelized solvers in cloud or high-performance computing environments, leveraging greater processing power.

The incorporation of these approaches could significantly reduce solution times, making the model more attractive for large-scale deployment across school networks.

5 CONCLUSION

This study demonstrated that Mixed-Integer Linear Programming can be effectively applied to teacher workload allocation under the Brazilian new secondary education reform. The proposed model successfully balanced operational efficiency with the satisfaction of individual teacher preferences, leading to a significant reduction in required presence days and the elimination of operational conflicts, such as the shared use of the Physical Education court.

The main contribution of this research lies in providing a decision-support tool for school managers that aligns institutional requirements with teacher well-being. However, it is acknowledged that the application to a single school limits the generalization of results, and that computational time may become a challenge when scaling up to larger school networks.

Future research should explore the adoption of techniques to accelerate model solving, such as decomposition methods and hybrid algorithms combining MILP with heuristics. These directions may enable the model to evolve into a more scalable and widely applicable solution for the Brazilian educational system.

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