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POLYGONAL THEOREM BETWEEN TWO PARALLEL LINES

TEOREMA POLIGONAL ENTRE DUAS RETAS PARALELAS

TEOREMA POLIGONAL ENTRE DOS RECTAS PARALELAS

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ABSTRACT

The aim of this proposal is to investigate the theorem of the polygonal line defined by transversals constructed between two parallel lines. This is a related theorem to the alternate angles theorem, commonly known as the “Beaks Theorem”. A task has been created for a dynamic environment using the GeoGebra geometry application for smartphones. It is expected that students will formulate conjectures and develop a proof or persuasive methods for emerging ideas related to the theorem.

Keywords: Parallel Lines and Transversals. Polygonal Line. Dynamic Geometry Environment. GeoGebra App. Smartphones.

RESUMO

O objetivo desta proposta é investigar o teorema da linha poligonal definida por transversais construídas entre duas retas paralelas. Este é um teorema relacionado ao teorema dos ângulos alternos, comumente conhecido como “Teorema do Bico”. Uma tarefa foi criada para um ambiente dinâmico utilizando a aplicação de geometria GeoGebra para smartphones. Espera-se que os alunos formulem conjecturas e desenvolvam uma prova ou métodos persuasivos para ideias emergentes relacionadas ao teorema.

Palavras-chave: Retas Paralelas e Transversais. Linha Poligonal. Ambiente de Geometria Dinâmica. Aplicativo GeoGebra. Smartphones.

RESUMEN

El objetivo de esta propuesta es investigar el teorema de la línea poligonal definida por transversales construidas entre dos rectas paralelas. Este es un teorema relacionado con el teorema de ángulos alternos, comúnmente conocido como “Teorema del Pico”. Se creó una tarea para un entorno dinámico utilizando la aplicación de geometría GeoGebra para smartphones. Se espera que los estudiantes formulen conjeturas y desarrollen una prueba o métodos persuasivos para ideas emergentes relacionadas con el teorema.

Palabras clave: Rectas Paralelas y Transversales. Línea Poligonal. Ambiente de Geometría Dinámica. Aplicativo GeoGebra. Smartphones.

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1 INTRODUCTION

> (...) *If the proof starts from axioms, distinguish has several cases, and takes thirteen lines in the textbook. It may give the youngsters the impression that mathematics consists in proving the most obvious things in the least obvious way* (POLYA, 1962, p. 129).

The *smartphone* enables interaction through the exchange of messages, audio or video calls, access to digital content, financial transactions, *online* shopping, GPS[2], and more. These possibilities showcase the mathematical applications used in the development of devices and applications. However, mathematical concepts can also be explored from mobile devices with touches on screens (MDwTS), smartphone, tablet, iPad.

A variety of applications with different purposes, including constructions, games, puzzles and formula sheets, are available for exploring mathematical ideas. Particularly, when approaching topics in Geometry, one can use those that offer the ambience of dynamic geometry with the accomplishment of a wide range tasks.

In this context, this proposal focuses on presenting a task that considers MDwTS, the theme, methods of manipulations (constructions), and opportunities for interactions and reflections were considered. Therefore, the objective is to investigate the “Beaks Theorem”, which is closely related to the relationships between angles obtained from the study of parallel lines intersected by a transversal, using the GeoGebra[3] geometry application.

2 PARALLEL LINES CUT BY A TRANSVERSAL

The study of Euclidean Geometry begins with the elementary figures in the plane: points and lines (Barbosa, 1985). It constitutes a mathematical system characterized by a unique vocabulary and properties, built upon undefined terms, from which the additional terms are established. As a result, the system's properties, including axioms and theorems, become apparent (Alexander; Koeberlein, 2010).

However, the geometric approach, through a dynamic environment such as GeoGebra, often bypasses this structure, assuming that the application is inherently grounded in Euclidean logic (Henrique; Bairral, 2022). For instance, to insert a **Line**, one must first construct two points. Similarly, the construction of a **parallel line** necessitates the prior creation of a line. According to the information provided by the GeoGebra Geometry app (version: 5.0.751.0) when we select the two tools:

[2] Global Positioning System.
[3] Available at: <https://www.geogebra.org/download. Access>. in: 04 set. 2023.

- **Line**: “select two points or two positions”.
- **Parallel line**: “select the point first, and then the line (or segment, or semi-straight or vector).”

Thus, by focusing on a central theme and utilizing a dynamic geometry environment, it becomes possible to build and develop concepts that form the foundation of Plane Euclidean Geometry. As demonstrated by Henrique (2021), exploring the postulate of corresponding angles and related theorems in a dynamic environment with MDwTS can fulfill this role. Investigating the properties of angles formed by two parallel lines intersected by a transversal enables a revisit of fundamental concepts such as the notion of parallel lines, angles, properties of concurrent lines, and advancement in the study of theorems related to triangles and quadrilaterals.

This topic forms the basis of a significant portion of geometric content covered in the final years of elementary school and, often, in textbooks, is presented in a superficial manner with a focus on applying algebraic techniques and procedures rather than embracing its geometric essence. Such an approach may undermine the essential role that Geometry plays in cognitive development and the cultivation of a range of skills, for example: imagination, intuition and visualization, all which contribute to the construction and conceptual development (Henrique, 2022).

Due to the relevance of this theme in building fundamental concepts and in geometric exploration, we propose a research project utilizing the GeoGebra geometry application for smartphones.

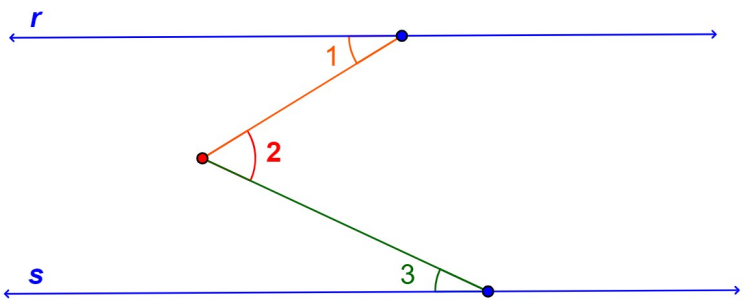
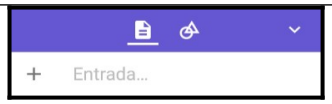
3 CONJECTURING THE BEAKS THEOREM

The property that relates angles in a polygonal line resulting from two parallel lines, also known as the “Beaks Theorem”, establishes a relationship between the angles formed. As a proposal for investigation in the classroom through the GeoGebra[4] application, we present the task “Angles in a Polygonal” (Table 1) in which nature combines the ubiquity of MDwTS and the dynamism of the application, which allows subjects to visualize varied forms of the mathematical object, formulate conjectures and construct proofs.

[4] For this purpose the application must be installed on the *students' own smartphones*. Although it is not the objective of this planning, we clarify that this type of activity requires teacher organization, including a moment of setting for students to become familiar with the use of the application. For more details, see Bairral and Henrique (2021).

Table 1

Construction of segments between parallel lines

Task: Angles in a polygon	GeoGebra Tools
Construct two parallel lines and two segments (polygonal line), as shown in the figure.	Line Parallel Line Segment Angle Move
	
Build angles 1, 2, and 3. In the algebra field under **Input** add 1 and 3 and compare the result with the angle measure 2, then freely move the lines and segments in order to modify the angles.	
What is the relationship between the angles? Draw up a conjecture and justify it.	

Source: own elaboration.

3.1 EXPLANATION

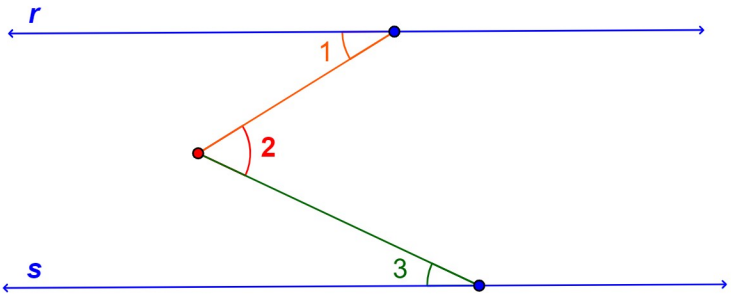
Students are expected to verify that the sum of angles 1 and 3 is equal to the measure of angle 2. This relation represents an immediate application of postulate (1) below and related theorems 1.1. and 1.2.:

1. If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
1. If two parallel lines are cut by a transversal, then the alternating angles are congruent.
1. If two parallel lines are cut by a transversal, then the collateral angles are supplemental.

The sum of angles 1 and 3 equals to the measure of $\angle$ 2 (Figure 1). The construction of the proof for this fact, performed with the GeoGebra app, can occur as follows: construct line **t,** parallel to lines **r** and **s**, starting from the point that contains the vertex of the $\angle$ 2 (Figure 2). Consequently, the angle that forms the “beak” ($\angle$ 2, Figure 1) is divided by the line **t** into angles 1 and 3 (Figure 2). Therefore, from theorem 1.1., one has that the angle of the “beak” is equal to the sum of angles 1 and 3.

Figure 1

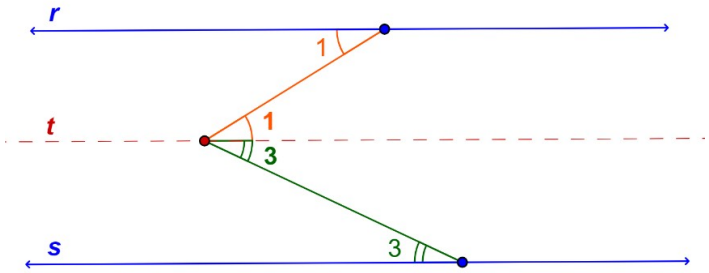
Angles in a polygonal line intersected by two parallel lines



Source: own elaboration.

Figure 2

Proof construction process



Source: own elaboration.

The results obtained in this process can be generalized, as the relationship will be verified regardless of the number of “beaks”.

The proposed task can be supplemented with other activities, such as synthesizing the ideas through writing and problem-solving[5]. We agree with Stein and Smith (2009) when they assert that it is the cumulative effect of different types of tasks in the classroom that allows mathematical ideas to mature.

Therefore, to summarize the results obtained, one can work on the preparation of a report which, according to Ponte, Brocardo and Oliveira (2006), involves the creation of a concise discursive text, in which the aim is to engage students in reflective action by presenting the results and conclusions regarding the activities, while also serving as an evaluative tool for the teacher. In the spirit of encouraging reflection, the proposed problems can encompass an analysis that is strictly written or assisted by applications for generalizing results.

[5] The notion of a problem here is based on Polya's definition (2006) of a determination problem, which can be either theoretical or concrete, with the purpose of finding the unknown by involving various procedures (calculating, writing, drawing, etc.).

4 CONCLUSION

The “Beak Theorem” should be used to engage upper-level elementary school students in formulating conjectures and constructing the process of proof or emerging forms of persuasion in a dynamic geometry environment with installed on students' smartphones. This proposal also allows for supplementation with activities that involve various forms of documentation, such as text production and solving other tasks, as well as different teaching methodologies, such as problem-solving.

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